PHYS 231 - Oct. 18, 2023

Last Time:

Euler's Eq'n: $e^{\pm j\phi} = \cos\phi \pm j\sin\phi$

Complex nos. can be expressed as:

$$z = x + jy = |z|(\cos\phi + j\sin\phi) = |z|e^{j\phi}$$

$$|z| = \sqrt{x^2+y^2} \qquad x = |z|\cos\phi$$

$$\tan\phi = \frac{y}{x} \qquad y = |z|\sin\phi$$

Complex Conjugate:

$$z^* = x - jy = |z|(\cos\phi - j\sin\phi)$$

$$= |z|e^{-j\phi}$$

Why define the complex conjugate of z?
$\rightarrow$ It can be used to find the magnitude of z.

Consider $z = x + jy = |z| e^{j\phi}$

then $z^* = x - jy = |z| e^{-j\phi}$

$$\therefore \quad z z^* = (x + jy)(x - jy)$$

$$= x^2 - j\cancel{x}y + j\cancel{x}y - \underbrace{j^2}_{+1} y^2$$

$$z z^* = x^2 + y^2 = |z|^2$$

$$\boxed{\therefore \quad |z| = \sqrt{z z^*}}$$

z times its conjugate z^* __must__ result in a purely real number $\rightarrow$ no j's.

Try $z z^* = \left(|z|\cancel{e^{j\phi}}\right)\left(|z|\cancel{e^{-j\phi}}\right)$

$= |z|^2$ ✓ same as above.

<u>Example:</u>

Suppose $z = \dfrac{1}{3+2j}$

Express z in the form:

(a) $z = x + jy \rightarrow$ find x & y

(b) $z = |z|e^{j\phi} \rightarrow$ find $|z|$ & ϕ

(a) Strategy to remove "j's" from btm is to mult. top & btm by complex conjugate (c.c.) of the btm.

$$z = \frac{1}{3+2j}\,\frac{(3-2j)}{(3-2j)} = \frac{3-2j}{3^2 + \cancel{6j} - \cancel{6j} \cancel{-} \overset{1}{j^2}2^2}$$

$$z = \frac{3-2j}{13} = \frac{3}{13} - \frac{2}{13}j$$

$$x = \frac{3}{13} \qquad y = -\frac{2}{13}$$

(b)

$$|z|^2 = x^2 + y^2 = \frac{3^2}{13^2} + \frac{2^2}{13^2}$$

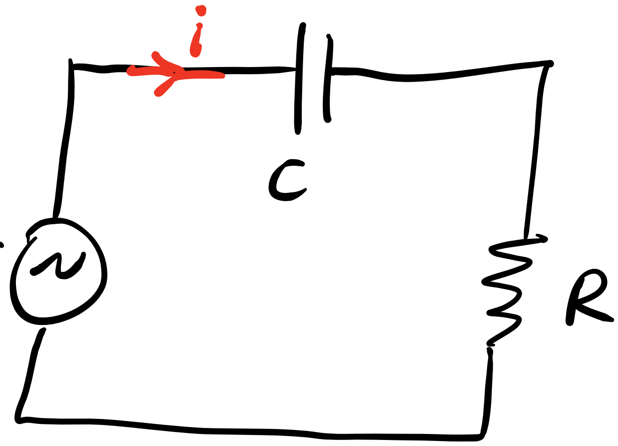
$$= \frac{\cancel{13}}{13^{\cancel{2}}} = \frac{1}{13}$$

$$\boxed{\therefore |z| = \frac{1}{\sqrt{13}}}$$

$$\tan\phi = \frac{y}{x} = \frac{-2/\cancel{13}}{3/\cancel{13}} \Rightarrow \tan\phi = -\frac{2}{3}$$

$$\boxed{\phi = \tan^{-1}\left(-\frac{2}{3}\right) = -\tan^{-1}\left(\frac{2}{3}\right)}$$

Previously solver for the current in the following RC circuit:



$v_{in} = V_0 \sin\omega t$

if $i = I_0 \sin(\omega t + \phi)$

$$I_0 = \frac{\omega V_0 C}{\sqrt{1+(\omega RC)^2}}$$

$$\tan\phi = \frac{1}{\omega RC}$$

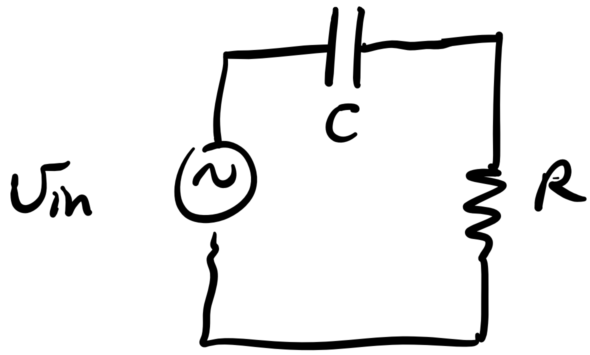
Suppose, for convenience, we express v_{in} as $v_{in} = V_0 e^{j\omega t}$

Then $v_{in} = V_0 e^{j\omega t} = V_0(\cos\omega t + j\sin\omega t)$

If we take the imaginary part, recover the original form of v_{in}.

$$\mathrm{Im}[v_{in}] = V_0 \sin\omega t$$

→ do calc. using complex exponential, can recover correct forms for results if we take the Im component in the end.



Loop Rule:

$$V_{in} - V_C - V_R = 0$$

$$V_{in} = \frac{q}{C} + iR$$

Take time deriv. of this expression

$$\underline{\frac{dV_{in}}{dt}} = \frac{1}{C}\underline{i} + R\,\underline{\frac{di}{dt}}$$

Know that $\frac{dV_{in}}{dt} = \frac{d}{dt}\left(V_0 e^{j\omega t}\right)$

$$= \underline{j\omega V_0 e^{j\omega t}}$$

Like we did for V_{in}, express i as complex exponential:

$$i = I_0 e^{j(\omega t + \phi)} = \underline{I_0 e^{j\omega t} e^{j\phi}}$$

Then $\frac{di}{dt} = \underline{j\omega I_0 e^{j\omega t} e^{j\phi}}$

subbing in the results, we have:

$$\text{(A)} \quad j\omega V_0 \cancel{e^{j\omega t}} = \frac{I_0}{C} \cancel{e^{j\omega t}} e^{j\phi} + j\omega R I_0 \cancel{e^{j\omega t}} e^{j\phi}$$

$$j\omega V_0 = I_0 e^{j\phi}\left(\frac{1}{C} + j\omega R\right)$$

We're trying to find I_0 & ϕ, so solve for $I_0 e^{j\phi}$

$$I_0 e^{j\phi} = \frac{j\omega V_0}{\frac{1}{C} + j\omega R} = \underbrace{\frac{j\omega V_0 C}{1 + j\omega RC}}_{\equiv Z}$$

Remove j's from demon. of Z.

$$I_0 e^{j\phi} = \frac{j\omega V_0 C}{1 + j\omega RC} \frac{(1 - j\omega RC)}{(1 - j\omega RC)}$$

$$I_0 e^{j\phi} = \frac{\omega V_0 C (j + \omega RC)}{1 + (\omega RC)^2} = x + jy$$

$$x = \frac{\omega V_0 C \ \omega RC}{1 + (\omega RC)^2} \qquad y = \frac{\omega V_0 C}{1 + (\omega RC)^2}$$

$$|z| = \sqrt{x^2 + y^2} = \frac{\omega V_0 C}{1 + (\omega RC)^2} \sqrt{(\omega RC)^2 + 1}$$

$$\therefore |z| = \frac{\omega V_0 C}{\sqrt{1 + (\omega RC)^2}}$$

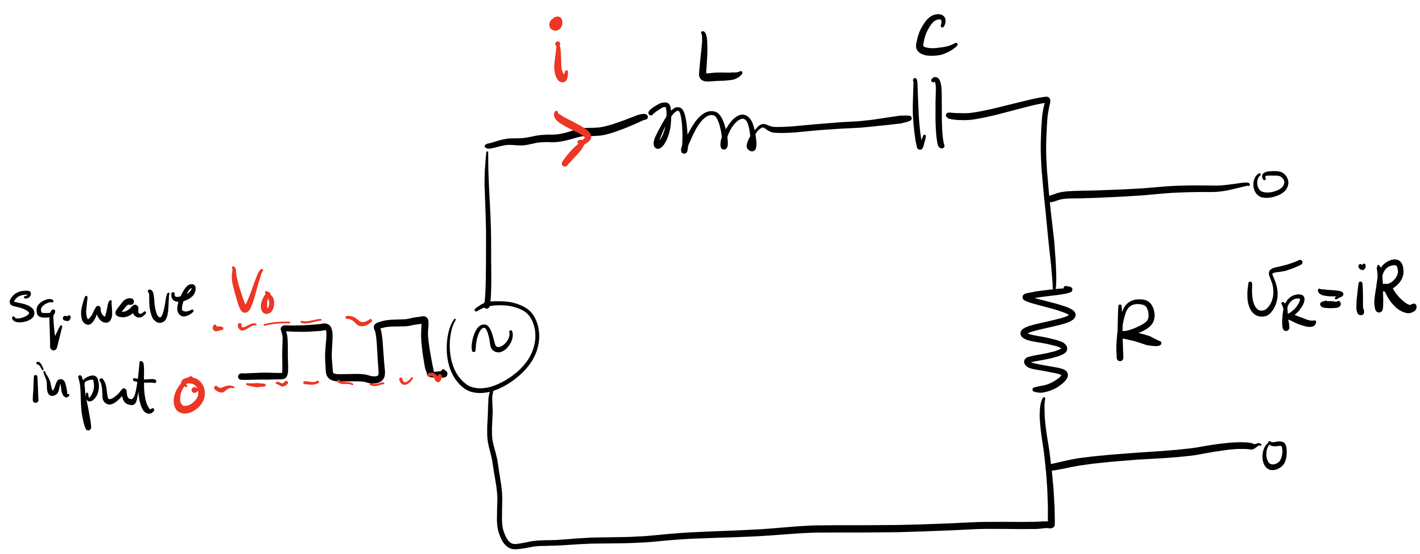
$$\tan \phi = \frac{y}{x} = \frac{1}{\omega RC}$$

$$\therefore I_0 e^{j\phi} = \underbrace{\frac{\omega V_0 C}{\sqrt{1 + (\omega RC)^2}}}_{I_0} e^{j\phi}$$

$$\phi = \tan^{-1}\left(\frac{1}{\omega RC}\right)$$

Experiment #4 - Day

LRC circuit transients



From Sept. 27, 2023 notes...

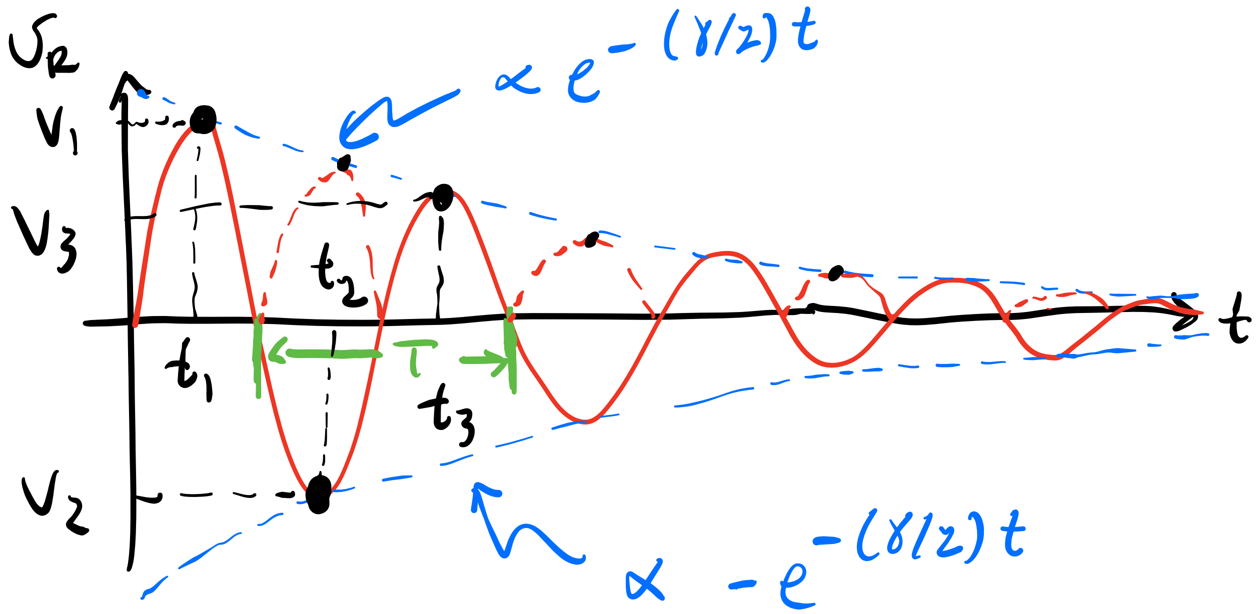
For an underdamped circuit in which

$$\omega_0 \gg \frac{\gamma}{2}$$

where $\omega_0 = \frac{1}{\sqrt{LC}}$ & $\gamma = \frac{R}{L}$

$$i \approx \omega_0 C V_0 e^{-(\gamma/2)t} \sin(\omega_0 t)$$

$$\therefore\ V_R = iR \approx \omega_0 RCV_0\, e^{-(\gamma/2)t} \sin(\omega_0 t)$$



meas. $\omega_0 = \frac{2\pi}{T} \Rightarrow$ compare to expected value of $\omega_0 = \frac{1}{\sqrt{LC}}$

Finding γ.

time	$\lvert V_R \rvert_{min,\, max}$
t_1	$V_1 \pm \Delta V_1$
t_2	$\lvert V_2 \rvert \pm \Delta V_2$
t_3	$V_3 \pm \Delta V_3$
t_4	$\lvert V_4 \rvert \pm \Delta V_4$
$\vdots$	$\vdots$
t_N	$\lvert V_N \rvert \pm \Delta V_N$

$$V_{meas} = V_0 e^{-(\gamma/2)t}$$

$$\underbrace{\ln V_{meas}}_{y} = \underbrace{\ln V_0}_{b} + \underbrace{-\left(\frac{\gamma}{2}\right)}_{m} \underbrace{t}_{x}$$

Plot $\ln V_{meas}$ vs t

slope $m = -\frac{\gamma}{2}$

Find $\gamma = -2m$

use Δm to also find $\Delta\gamma$

compare meas. γ to expected value of $\frac{R}{L}$.